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The influence of heat resistance and heat leak on the performance of a four-heat-reservoir absorption refrigerator with heat transfer law of $Q \propto \Delta(T^{-1})$

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Abstract

On the basis of an endoreversible absorption refrigeration cycle model with Newton's heat transfer law, an irreversible four-heat-reservoir cycle model with another linear heat transfer law of $Q \propto \Delta(T^{-1})$ is built by taking account the heat leak and heat resistance losses. The fundamental optimal relation between the coefficient of performance (COP) and the cooling load, the maximum COP and the corresponding cooling load, as well as the maximum cooling load and the corresponding COP of the cycle with another linear heat transfer law coupled to constant-temperature heat reservoirs are derived by using finite-time thermodynamics. The optimal distribution relation of the heat-transfer surface areas is also obtained. Moreover, the effects of the cycle parameters on the COP and the cooling load of the cycle are studied by detailed numerical examples. The results obtained herein are of importance to the optimal design and performance improvement of a four-heat-reservoir absorption refrigeration cycle.

Keywords: Finite time thermodynamics; Thermodynamic optimization; Four-heat-reservoir absorption refrigerator; Linear phenomenological heat transfer law

1. Introduction

The absorption refrigerators can be driven by 'low-grade' heat energy such as waste heat in industries, solar energy and geothermal energy, and have a large potential for reducing the heat pollution for the environment. Thus, absorption refrigerators for industrial and domestic use are generating renewed interest throughout the world. In the last years, finite-time thermodynamics [1–6] was applied to the performance study of absorption refrigerators [7–20], and a lot of results, which are different from those by using the classical thermodynamics, have been obtained. Yan et al. [8–10], Goktun [11], Bejan et al. [12,13] and Chen et al. [14] analyzed the performance of the three-heat-reservoir endoreversible [8–10,12,13] and irreversible [11,14] absorption refrigeration cycles with Fourier's heat transfer law [21]. Chen et al. [15–17] studied the performance of the three-heat-reservoir endoreversible [15,16] and irreversible [17] absorption refrigeration cycles with another linear heat transfer law, i.e., linear phenomenological heat transfer law, $Q \propto \Delta(T^{-1})$. A three-heat-reservoir absorption refrigerator is a simplified model that the temperature of a condenser is equal to that of an absorber, but a real absorption refrigerator is not. Therefore, a four-heat-reservoir absorption refrigeration cycle model is closer to a real absorption refrigerator. The performance of the four-heat-reservoir absorption refrigeration cycle with Fourier's heat transfer law was studied by Chen [18], Shi et al. [19] and Zheng et al. [20]. On the basis of these research work, a four-heat-reservoir absorption refrigeration cycle model with linear phenomenological heat transfer law, which included the heat leak from the heat sink to the cooled space and the finite-rate heat transfer between the working fluid and the external heat reservoirs, is established in this paper. The fundamental optimal relation between the COP and the cooling load, the maximum COP and the corresponding cooling load, as well as the maximum cooling load and the corresponding COP of the cycle cou-

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Nomenclature

a	distribution rate of the total heat reject quantity between the condenser and the absorber	
A	total heat-transfer surface area of the heat exchangers	m^2
A_1	heat-transfer surface area of the generator	m^2
A_2	heat-transfer surface area of the evaporator	m^2
A_3	heat-transfer surface area of the condenser	m^2
A_4	heat-transfer surface area of the absorber	m^2
K_L	heat leak coefficient	kW·K
Q_L	rate of heat leak	kW
Q_1	rate of heat absorbed from heat source	kW
Q_2	rate of heat absorbed from cooled space	kW
Q_3	rate of heat released from the condenser to heat sink	kW
Q_4	rate of heat released from the absorber to heat sink	kW
R	cooling load	kW
R_{max}	maximum cooling load	kW
R_ε	cooling load at the maximum COP	kW
T_H	temperature of heat source	K
T_M	temperature of heat sink	K
T_L	temperature of cooled space	K
T_O	temperature of heat sink	K
T_1	temperature of working fluid in the generator	K
T_2	temperature of working fluid in the evaporator	K
T_3	temperature of working fluid in the condenser	K
T_4	temperature of working fluid in the absorber	K
U_1	heat-transfer coefficients of the generator	$kW·K·m^{-2}$
U_2	heat-transfer coefficients of the evaporator	$kW·K·m^{-2}$
U_3	heat-transfer coefficients of the condenser	$kW·K·m^{-2}$
U_4	heat-transfer coefficients of the absorber	$kW·K·m^{-2}$
U	total heat exchanger inventory	kW·K

Greek symbols

τ	cycle period
τ_1	ratio of the temperature of the condenser-side heat sink and the evaporator-side heat source
τ_2	ratio of the temperatures of the generator-side heat source and the absorption-side heat sink
ε	COP
ε_{max}	maximum COP
ε_R	COP at the maximum cooling load

pled to constant-temperature heat reservoirs are derived. The results can provide the theoretical bases for the optimal design and operation of real absorption refrigeration operating at four temperature levels.

2. Physical model

A four-heat-reservoir irreversible absorption refrigeration cycle with linear phenomenological heat transfer law that consists of a generator, an evaporator, an absorber and a condenser, is shown in Fig. 1. The flow of the working fluid in the cycle system is stable and the different parts of the working fluid exchange heat with the heat reservoirs at temperature T_H, T_L, T_O and T_M during the full time τ, whereas there are thermal resistances between the working fluid and the external heat reservoirs. Therefore, the corresponding working fluid temperatures are T_1, T_2, T_3 and T_4, respectively. The cooling load of absorption refrigerators is always smaller than Q_2 because of the heat leak from the environmental reservoir to the cooled space. Work input required by the solution pump in the system is negligible relative to the energy input to the generator and is often neglected for the purpose of analysis. From the first law of thermodynamics and the second law of thermodynamics, one has $Q_1/T_1 + Q_2/T_2 - Q_3/T_3 - Q_4/T_4 = 0$ and $Q_1 + Q_2 - Q_3 - Q_4 = 0$. It is assumed that the heat transfers between the working fluid in the heat exchangers and the external heat reservoirs are carried out under a finite temperature difference and obey linear phenomenological heat transfer law, and these heat exchange processes are isothermal and the equations of heat transfer may be written as

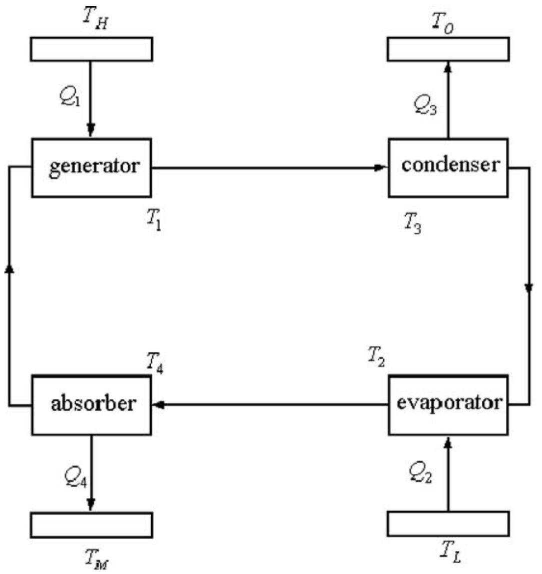
$$Q_1 = U_1 A_1 (T_1^{-1} - T_H^{-1})\tau$$
$$Q_2 = U_2 A_2 (T_2^{-1} - T_L^{-1})\tau$$
$$Q_3 = U_3 A_3 (T_O^{-1} - T_3^{-1})\tau$$
$$Q_4 = U_4 A_4 (T_M^{-1} - T_4^{-1})\tau$$
$$Q_L = K_L (2T_L^{-1} - T_M^{-1} - T_O^{-1})\tau \quad (1)$$

where U_1, U_2, U_3 and U_4 are, respectively, the overall heat-transfer coefficients of the generator, evaporator, condenser and absorber; K_L is the heat leak coefficient, and A_1, A_2, A_3 and A_4 are, respectively, the heat-transfer surface areas of the generator, evaporator, condenser and absorber. The total heat-transfer surface area is

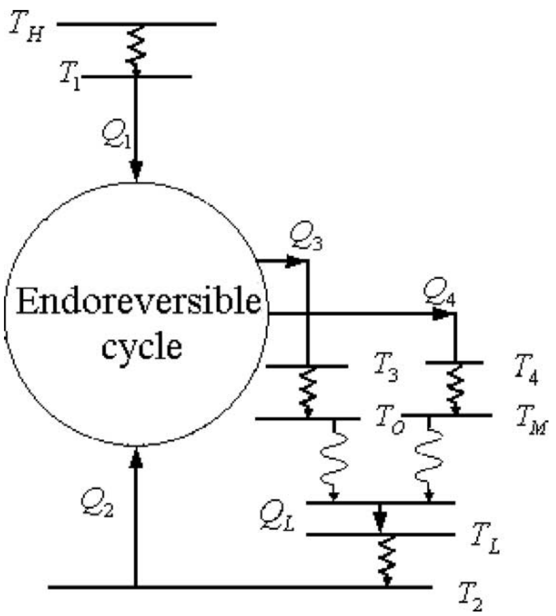
$$A = A_1 + A_2 + A_3 + A_4 \quad (2)$$

Defining the parameter a (the distribution rate of the total heat reject quantity between the condenser and the absorber):

$$a = Q_4/Q_3 \quad (3)$$



(a)



(b)

Fig. 1. A four-heat-reservoir irreversible absorption cycle model.

3. Fundamental optimal relation

According to the standard definitions of the COP (ε) and the cooling load (R) of an absorption refrigerator, one obtains

$$\begin{aligned}
\varepsilon &= \frac{R}{Q_1} = \frac{Q_2 - Q_L}{Q_1} = \frac{Q_2}{Q_1}\left(1 - \frac{Q_L}{Q_2}\right) \\
&= \frac{T_3^{-1} + aT_4^{-1} - (1+a)T_1^{-1}}{(1+a)T_2^{-1} - T_3^{-1} - aT_4^{-1}} \\
&\quad \times \left\{1 - \frac{q_L}{A}\left\{\left[(1+a)T_2^{-1} - T_3^{-1} - aT_4^{-1}\right]\right.\right. \\
&\quad \times \left[U_1\left(T_1^{-1} - T_H^{-1}\right)\left[T_3^{-1} + aT_4^{-1} - (1+a)T_1^{-1}\right]\right]^{-1} \\
&\quad + \frac{1}{U_2(T_2^{-1} - T_L^{-1})} \\
&\quad + \frac{T_2^{-1} - T_1^{-1}}{U_3(T_O^{+1} - T_3^{-1})[T_3^{-1} + aT_4^{-1} - (1+a)T_1^{-1}]} \\
&\quad \left.\left. + \frac{a(T_2^{-1} - T_1^{-1})}{U_4(T_M^{-1} - T_4^{-1})[T_3^{-1} + aT_4^{-1} - (1+a)T_1^{-1}]}\right\}\right\}
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
R &= \frac{Q_2 - Q_L}{\tau} \\
&= A\left\{\frac{(1+a)T_2^{-1} - T_3^{-1} - aT_4^{-1}}{U_1(T_1^{-1} - T_H^{-1})[T_3^{-1} + aT_4^{-1} - (1+a)T_1^{-1}]}\right. \\
&\quad + \frac{1}{U_2(T_2^{-1} - T_L^{-1})} \\
&\quad + \frac{T_2^{-1} - T_1^{-1}}{U_3(T_O^{-1} - T_3^{-1})[T_3^{-1} + aT_4^{-1} - (1+a)T_1^{-1}]} \\
&\quad \left. + \frac{a(T_2^{-1} - T_1^{-1})}{U_4(T_M^{-1} - T_4^{-1})[T_3^{-1} + aT_4^{-1} - (1+a)T_1^{-1}]}\right\}^{-1} \\
&\quad - q_L
\end{aligned} \tag{5}$$

where $q_L = Q_L/\tau$. For the sake of convenience, let $b_1 = T_3^{-1}$, $b_2 = (1+a)T_1^{-1}$, $b_3 = (1+a)T_2^{-1}$, $b_4 = aT_4^{-1}$, $C = q_L/A$ and $C = q_L/A$. Then, Eqs. (4) and (5) may be rewritten as

$$\begin{aligned}
\varepsilon &= \frac{b_1 + b_4 - b_2}{b_3 - b_1 - b_4} \\
&\quad \times \left\{1 - C\left[\frac{b_3 - b_1 - b_4}{U_1(\frac{b_2}{1+a} - T_H^{-1})(b_1 + b_4 - b_2)}\right.\right. \\
&\quad + \frac{1}{U_2(\frac{b_3}{1+a} - T_L)} \\
&\quad + \frac{b_3 - b_2}{U_3(1+a)(T_O^{-1} - b_1)(b_1 + b_4 - b_2)} \\
&\quad \left.\left. + \frac{a(b_3 - b_2)}{U_4(T_M^{-1} - \frac{b_4}{a})(b_1 + b_4 - b_2)}\right]\right\}
\end{aligned} \tag{6}$$

$$\begin{aligned}
R &= A\left[\frac{b_3 - b_1 - b_4}{U_1(\frac{b_2}{1+a} - T_H^{-1})(b_1 + b_4 - b_2)} + \frac{1}{U_2(\frac{b_3}{1+a} - T_L)}\right. \\
&\quad + \frac{b_3 - b_2}{U_3(1+a)(T_O^{-1} - b_1)(b_1 + b_4 - b_2)} \\
&\quad \left. + \frac{a(b_3 - b_2)}{U_4(T_M^{-1} - \frac{b_4}{a})(b_1 + b_4 - b_2)}\right]^{-1} - q_L
\end{aligned} \tag{7}$$

3.1. *The maximum COP and the corresponding cooling load*

Using Eq. (6) and the extremal conditions $\partial\varepsilon/\partial b_1 = 0$, $\partial\varepsilon/\partial b_2 = 0$, $\partial\varepsilon/\partial b_3 = 0$ and $\partial\varepsilon/\partial b_4 = 0$, one can derive the temperatures of the working fluid in the generator, absorber, condenser and evaporator which correspond the maximum COP $\varepsilon_{\max}$ as follows

$$
\begin{aligned}
T_1 &= T_{1\varepsilon} = 1/\left[T_H^{-1} + x\left(T_M^{-1} - B\right)\right] \\
T_2 &= T_{2\varepsilon} = 1/\left[T_L^{-1} + y\left(T_M^{-1} - B\right)\right] \\
T_3 &= T_{3\varepsilon} = 1/\left[T_O^{-1} - z\left(T_M^{-1} - B\right)\right] \\
T_4 &= T_{4\varepsilon} = 1/B
\end{aligned} \tag{8}
$$

where

$$
B = \frac{(1+a)U_4 T_M^{-1} C^{-1} - [y(1+a) + a + z] - \sqrt{B_1}}{(1+a)U_4 C^{-1}}
$$

$$
B_1 = \left[y(1+a) + a + z\right]^2 - (1+a)U_4\left(aT_M^{-1} + T_O^{-1}\right)C^{-1} + (1+a)^2 U_4 T_L^{-1} C^{-1}
$$

$$
x = (U_4/U_1)^{1/2}, \qquad y = (U_4/U_2)^{1/2}
$$
$$
z = (U_4/U_3)^{1/2}
$$

Substituting Eq. (8) into Eqs. (4) and (5), one obtain the maximum COP

$$
\varepsilon_{\max} = \frac{D_1}{D_2}\left[1 - \frac{q_L}{A} \times \frac{(1+a)(xD_2 + yD_1) + (z+a)D_3}{(1+a)D_1 U_4 (T_M^{-1} - B)}\right] \tag{9}
$$

and the corresponding cooling load

$$
R_\varepsilon = \frac{A(1+a)D_1 U_4 (T_M^{-1} - B)}{(1+a)(xD_2 + yD_1) + (z+a)D_3} - q_L \tag{10}
$$

where

$$
\begin{aligned}
D_1 &= aB + T_O^{-1} - (1+a)T_H^{-1} - \left(T_M^{-1} - B\right)\left[z + (1+a)x\right] \\
D_2 &= (1+a)T_L^{-1} - aB - T_O^{-1} + \left(T_M^{-1} - B\right)\left[y(1+a) + z\right] \\
D_3 &= D_1 + D_2
\end{aligned}
$$

$\varepsilon_{\max}$ and R_ε are two important parameters of absorption refrigerators. They determine the upper bound of the COP and the lower bound of the cooling load.

Using Eqs. (1) and (8) one can obtain the optimal distribution relation of the heat-transfer surface areas as follows

$$
\frac{A_1}{A_2} = \frac{xD_2}{yD_1}, \qquad \frac{A_3}{A_4} = \frac{z}{a}
$$

$$
\begin{aligned}
\frac{A_1 + A_2}{A_3 + A_4}
&= \left\{(1+a)\left(xT_L^{-1} - yT_H^{-1}\right) + (y-x)\left[T_O^{-1} + aB - z\left(T_M^{-1} - B\right)\right]\right\} \\
&\quad \times \left\{(z+a)\left[T_L^{-1} - T_H^{-1} + (y-x)\left(T_M^{-1} - B\right)\right]\right\}^{-1}
\end{aligned} \tag{11}
$$

From Eqs. (2) and (11), one can find the relations between the heat-transfer surface areas and the total heat-transfer surface area A as follows

$$
\begin{aligned}
A_1 = xD_2A
&\times \left\{(1+a)\left(xT_L^{-1} - yT_H^{-1}\right) + (y-x)\left(T_O^{-1} + aB\right)\right. \\
&\left. + (z+a)\left(T_L^{-1} - T_H^{-1}\right) + a(y-x)\left(T_M^{-1} - B\right)\right\}^{-1}
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
A_2 = yD_1A
&\times \left\{(1+a)\left(xT_L^{-1} - yT_H^{-1}\right) + (y-x)\left(T_O^{-1} + aB\right)\right. \\
&\left. + (z+a)\left(T_L^{-1} - T_H^{-1}\right) + a(y-x)\left(T_M^{-1} - B\right)\right\}^{-1}
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
A_3 = zD_3A
&\times \left\{(1+a)\left[(1+a)\left(xT_L^{-1} - yT_H^{-1}\right)\right.\right. \\
&+ (y-x)\left(T_O^{-1} + aB\right) \\
&\left.\left. + (z+a)\left(T_L^{-1} - T_H^{-1}\right) + a(y-x)\left(T_M^{-1} - B\right)\right]\right\}^{-1}
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
A_4 = aD_3A
&\times \left\{(1+a)\left[(1+a)\left(xT_L^{-1} - yT_H^{-1}\right)\right.\right. \\
&+ (y-x)\left(T_O^{-1} + aB\right) \\
&\left.\left. + (z+a)\left(T_L^{-1} - T_H^{-1}\right) + a(y-x)\left(T_M^{-1} - B\right)\right]\right\}^{-1}
\end{aligned} \tag{15}
$$

3.2. *The maximum cooling load and the corresponding COP*

Using Eq. (7) and the extremal conditions $\partial R/\partial b_1 = 0$, $\partial R/\partial b_2 = 0$, $\partial R/\partial b_3 = 0$ and $\partial R/\partial b_4 = 0$, one can derive the temperatures of the working fluid in the generator, absorber, condenser and evaporator which the maximum $R_{\max}$ as follows

$$
\begin{aligned}
T_1 &= T_{1R} = 1/\left[T_H^{-1} + x\left(T_M^{-1} - E\right)\right] \\
T_2 &= T_{2R} = 1/\left[T_L^{-1} + y\left(T_M^{-1} - E\right)\right] \\
T_3 &= T_{3R} = 1/\left[T_O^{-1} - z\left(T_M^{-1} - E\right)\right] \\
T_4 &= T_{4R} = 1/E
\end{aligned} \tag{16}
$$

where

$$
E = \frac{[x(1+a) + a + 2z]T_M^{-1} + (1+a)T_H^{-1} - T_O^{-1}}{2[x(1+a) + a + z]}
$$

Substituting Eq. (16) into Eqs. (4) and (5) yields the maximum cooling load

$$
R_{\max} = \frac{A(1+a)F_1 U_4 (T_M^{-1} - E)}{(1+a)(xF_2 + yF_1) + (z+a)F_3} - q_L \tag{17}
$$

and the corresponding COP

$$
\varepsilon_R = \frac{F_1}{F_2}\left[1 - \frac{q_L}{A} \times \frac{(1+a)(xF_2 + yF_1) + (z+a)F_3}{(1+a)F_1 U_4 (T_M^{-1} - E)}\right] \tag{18}
$$

where

$$F_1 = T_O^{-1} - (1+a)T_H^{-1} + aE - \big[(1+a)x + z\big]\big(T_M^{-1} - E\big)$$
$$F_2 = (1+a)T_L^{-1} - T_O^{-1} - aE + \big[(1+a)y + z\big]\big(T_M^{-1} - E\big)$$
$$F_3 = F_1 + F_2$$

Using Eqs. (1) and (16) one can obtain the optimal distribution relation of the heat-transfer surface areas as follows

$$\frac{A_1}{A_2} = \frac{xF_2}{yF_1}, \qquad \frac{A_3}{A_4} = \frac{z}{a}$$
$$\begin{aligned}\frac{A_1 + A_2}{A_3 + A_4} &= \big\{(1+a)\big(xT_L^{-1} - yT_H^{-1}\big) \\ &\quad + (y-x)\big[T_O^{-1} + aE - z\big(T_M^{-1} - E\big)\big]\big\} \\ &\quad \times \big\{(z+a)\big[T_L^{-1} - T_H^{-1} + (y-x)\big(T_M^{-1} - E\big)\big]\big\}^{-1}\end{aligned} \tag{19}$$

From Eqs. (2) and (19), one can find the relations between the heat-transfer surface areas and the total heat-transfer surface area A as follows

$$\begin{aligned}A_1 &= xF_2A \\ &\quad \times \big\{(1+a)\big(xT_L^{-1} - yT_H^{-1}\big) + (y-x)\big(T_O^{-1} - aE\big) \\ &\quad + (z+a)\big(T_L^{-1} - T_H^{-1}\big) + a(y-x)\big(T_M^{-1} - E\big)\big\}^{-1}\end{aligned} \tag{20}$$

$$\begin{aligned}A_2 &= yF_1A \\ &\quad \times \big\{(1+a)\big(xT_L^{-1} - yT_H^{-1}\big) + (y-x)\big(T_O^{-1} - aE\big) \\ &\quad + (z+a)\big(T_L^{-1} - T_H^{-1}\big) + a(y-x)\big(T_M^{-1} - E\big)\big\}^{-1}\end{aligned} \tag{21}$$

$$\begin{aligned}A_3 &= zF_3A \\ &\quad \times \big\{(1+a)\big[(1+a)\big(xT_L^{-1} - yT_H^{-1}\big) \\ &\quad + (y-x)\big(T_O^{-1} - aE\big) \\ &\quad + (z+a)\big(T_L^{-1} - T_H^{-1}\big) + a(y-x)\big(T_M^{-1} - E\big)\big]\big\}^{-1}\end{aligned} \tag{22}$$

$$\begin{aligned}A_4 &= aF_3A \\ &\quad \times \big\{(1+a)\big[(1+a)\big(xT_L^{-1} - yT_H^{-1}\big) \\ &\quad + (y-x)\big(T_O^{-1} - aE\big) \\ &\quad + (z+a)\big(T_L^{-1} - T_H^{-1}\big) + a(y-x)\big(T_M^{-1} - E\big)\big]\big\}^{-1}\end{aligned} \tag{23}$$

3.3. Fundamental optimal relation

Combining Eqs. (6), (7), and the extremal conditions $\partial\varepsilon/\partial b_1 = 0$, $\partial\varepsilon/\partial b_2 = 0$, $\partial\varepsilon/\partial b_3 = 0$, $\partial\varepsilon/\partial b_4 = 0$, $\partial R/\partial b_1 = 0$, $\partial R/\partial b_2 = 0$, $\partial R/\partial b_3 = 0$ and $\partial R/\partial b_4 = 0$ yields a general relation

$$\begin{aligned}U_1^{1/2}\big[b_2/(1+a) - T_H^{-1}\big] &= U_2^{1/2}\big[b_3/(1+a) - T_L^{-1}\big] \\ &= U_3^{1/2}\big(T_O^{-1} - b_1\big) = U_4^{1/2}\big(T_M^{-1} - b_4/a\big)\end{aligned} \tag{24}$$

One can prove further that Eq. (24) also holds for other optimum states of absorption refrigerators. Substituting the solutions of Eq. (24) into Eqs. (4) and (5) yields

$$\begin{aligned}\varepsilon &= \frac{T_O^{-1} - (1+a)T_H^{-1} + b_4 - [(1+a)x + z](T_M^{-1} - b_4/a)}{(1+a)T_L^{-1} - T_O^{-1} - b_4 + [(1+a)y + z](T_M^{-1} - b_4/a)} \\ &\quad \times \frac{R}{R + q_L}\end{aligned} \tag{25}$$

$$\begin{aligned}R &= \big\{U_4A(1+a)\big(T_M^{-1} - b_4/a\big)\big\{T_O^{-1} - (1+a)T_H^{-1} \\ &\quad + b_4 - \big[(1+a)x + z\big]\big(T_M^{-1} - b_4/a\big)\big\}\big\} \\ &\quad \times \big\{(1+a)\big(xT_L^{-1} - yT_H^{-1}\big) + (y-x)\big(T_O^{-1} - b_4\big) \\ &\quad + (z+a)\big(T_L^{-1} - T_H^{-1}\big) + (y-x)\big(aT_M^{-1} - b_4\big)\big\}^{-1} \\ &\quad - q_L\end{aligned} \tag{26}$$

Eliminating b_4 from Eqs. (25) and (26) yields a fundamental optimum relation of an irreversible absorption refrigerator, which can be used directly to analyze the influence of major irreversibilities on the performance of an irreversible absorption refrigerator and to plot the characteristic curve of an irreversible absorption refrigerator with linear phenomenological heat transfer law, as shown by curve a in Fig. 2. It exhibits a loop-shaped one. The characteristic curve of an endoreversible absorption refrigerator with the sole irreversibility of heat resistance is also illustrated, see curve b in Fig. 2. It is similar to a parabola. Fig. 2 shows the difference of the characteristic curves between an irreversible absorption refrigerator with heat resistance and heat leak losses and an endoreversible one with heat resistance loss. In the calculation, $A = 1100\ \mathrm{m}^2$, $T_O = 303$ K, $T_H = 403$ K, $T_M = 305$ K, $T_L = 273$ K, $a = 1.5$, $U_1 = 682.223\ \mathrm{kW{\cdot}K{\cdot}m^{-2}}$, $U_2 = 458.167\ \mathrm{kW{\cdot}K{\cdot}m^{-2}}$ $U_3 = 1622.5\ \mathrm{kW{\cdot}K{\cdot}m^{-2}}$, $U_4 = 1452.06\ \mathrm{kW{\cdot}K{\cdot}m^{-2}}$, $K_L = 104.284\ \mathrm{kW{\cdot}K}$ are set [7,22].

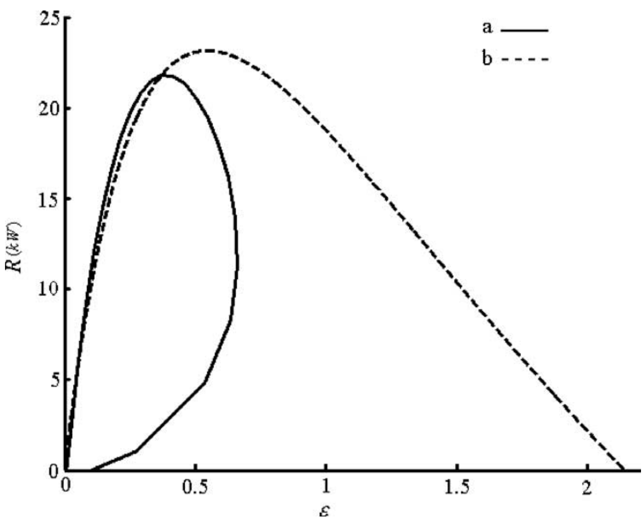


Fig. 2. The optimal COP ε versus the cooling load R.

4. Results and discussion

4.1. It is seen from Fig. 2 that the $R-\varepsilon$ characteristic curve of an irreversible absorption refrigerator with linear phenomenological heat transfer law is divided into three parts by the three operating states of $\varepsilon = 0$, $\varepsilon = \varepsilon_R$ and $\varepsilon = \varepsilon_{\max}$. When the absorption refrigerator is operated in those parts of the $R-\varepsilon$ curve that have a positive slope, the COP decreases as the cooling load decreases. These regions are not the optimal operating regions. The optimal operating region of an absorption refrigerator should be situated in that part of the $R-\varepsilon$ curve that has a negative slope. In such a case, the COP will increase as the cooling load decreases, and vice versa. Thus, the COP and cooling load should be, respectively, constrained by

$$\varepsilon_R \leqslant \varepsilon \leqslant \varepsilon_{\max}, \qquad R_{\max} \geqslant R \geqslant R_\varepsilon \tag{27}$$

According to Eq. (27), one can determine that the optimal regions of the temperatures of the working fluid in the heat exchangers are

$$T_{1\varepsilon} \geqslant T_1 \geqslant T_{1R}, \qquad T_{2\varepsilon} \geqslant T_2 \geqslant T_{2R}$$
$$T_{3R} \geqslant T_3 \geqslant T_{3\varepsilon}, \qquad T_{4R} \geqslant T_4 \geqslant T_{4\varepsilon} \tag{28}$$

4.2. In order to discuss the optimal performance, defining the ratio $\tau_1 = T_O/T_L$ of the temperature of the condenser and the evaporator, and the ratio $\tau_2 = T_H/T_M$ of the temperature of the generator and the absorption. One can analyze the performance numerically. In the calculation, the cycle parameters are set as same as stated above.

The influences of τ_1 on the maximum COP $\varepsilon_{\max}$ and the corresponding cooling load R_ε, the maximum cooling load $R_{\max}$ and the corresponding COP ε_R of the four-heat-reservoir irreversible absorption refrigeration cycle with linear phenomenological heat transfer law versus a with $T_H = 403$ K is shown in Figs. 3–6. Figs. 3–6 show that for a fixed a, $\varepsilon_{\max}$, R_ε, $R_{\max}$ and ε_R decrease with the increase of τ_1. When a is larger than one value, the influence of τ_1 on $\varepsilon_{\max}$, R_ε, $R_{\max}$ and ε_R are less. For a fixed a,

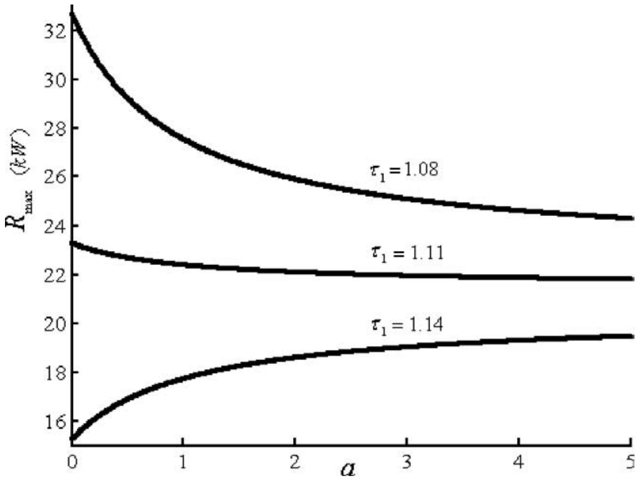


Fig. 3. The influence of τ_1 on the maximum COP $\varepsilon_{\max}$ versus a.

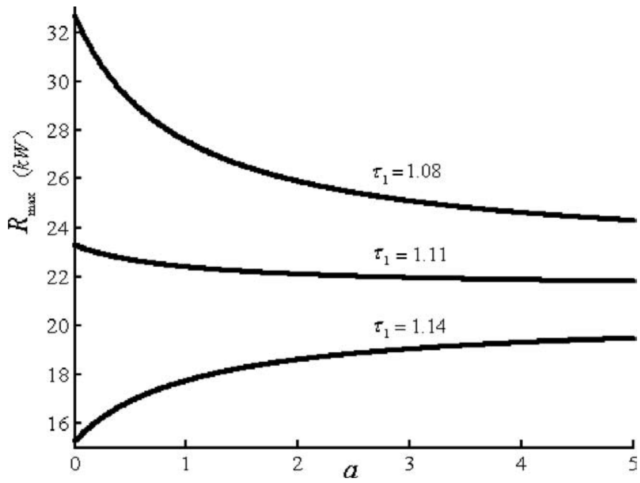


Fig. 5. The influence of τ_1 on the maximum cooling load $R_{\max}$ versus a.

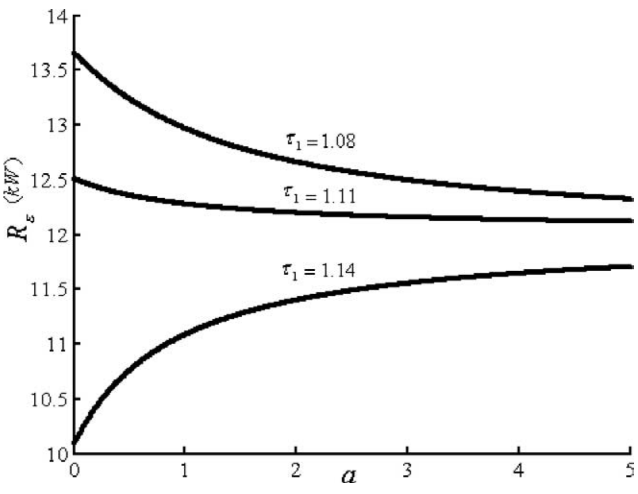


Fig. 4. The influence of τ_1 on the cooling load R_ε versus a.

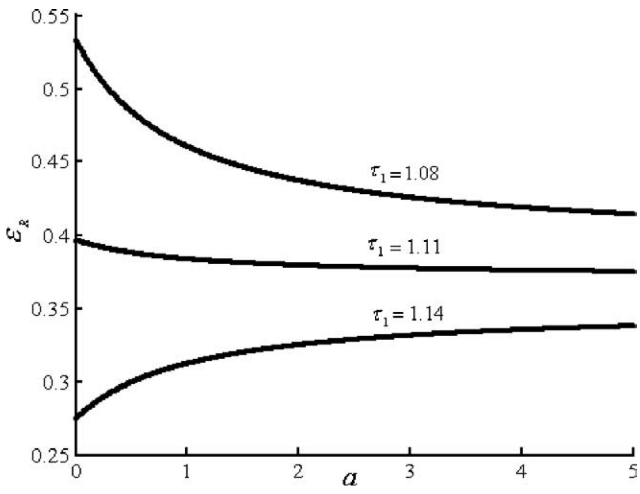


Fig. 6. The influence of τ_1 on the COP ε_R versus a.

there is an extremal point at $\tau_1 = 1.1172$ for the cycle. When $\tau_1 < 1.1172$, $\varepsilon_{\max} \geqslant 0.6708$, $R_\varepsilon \geqslant 12.3$, $R_{\max} \geqslant 22.8$, $\varepsilon_R \geqslant 0.3802$ and $\varepsilon_{\max}$, R_ε, $R_{\max}$ and ε_R decrease with the increase of a; and when $\tau_1 > 1.1172$, $\varepsilon_{\max} < 0.6708$, $R_\varepsilon < 12.3$, $R_{\max} < 22.8$, $\varepsilon_R < 0.3802$ and $\varepsilon_{\max}$, R_ε, $R_{\max}$ and ε_R increase with the increase of a. These imply that they will reach their asymptotic values, i.e., $\varepsilon_{\max} \to 0.6708$, $R_\varepsilon \to 12.3$, $R_{\max} \to 22.8$, $\varepsilon_R \to 0.3802$ when a tend to infinity.

The influences of τ_2 on the maximum COP $\varepsilon_{\max}$ and the corresponding cooling load R_ε, the maximum cooling load $R_{\max}$ and the corresponding COP ε_R of the four-heat-reservoir irreversible absorption refrigeration cycle with linear phenomenological heat transfer law versus a with $T_O = 313$ K is shown in Figs. 7–10. Figs. 7–10 show that for a fixed a, $\varepsilon_{\max}$, R_ε, $R_{\max}$ and ε_R increase with the increase of τ_2. Here $\tau_1 > 1.1172$ holds, $\varepsilon_{\max}$, R_ε, $R_{\max}$ and ε_R increase with the increase of a.

4.3. The performance optimization can be carried out by optimizing the distribution of the heat exchanger total inventory [12]. If one use $UA = U_1A_1 + U_2A_2 + U_3A_3 + U_4A_4$ to replace $A = A_1 + A_2 + A_3 + A_4$, i.e., using the distribution of the heat conductance to replace the distribution of the heat-transfer surface areas, one can obtain the optimal distribution relation of the heat conductance as follows

$$\frac{U_1A_1}{U_2A_2} = T_O^{-1} - (1+a)T_H^{-1} + b_4 - (2+a)\left(T_M^{-1} - \frac{b_4}{a}\right)$$

$$\frac{U_3A_3}{U_4A_4} = \frac{1}{a}$$

$$U_1A_1 + U_2A_2 = U_3A_3 + U_4A_4 = \frac{UA}{2} \tag{29}$$

and the cooling load and the COP are

$$\varepsilon = \frac{T_O^{-1} - (1+a)T_H^{-1} + b_4 - (2+a)(T_M^{-1} - b_4/a)}{(1+a)T_L^{-1} - T_O^{-1} - b_4 + (2+a)(T_M^{-1} - b_4/a)} \times \frac{R}{R + q_L} \tag{30}$$

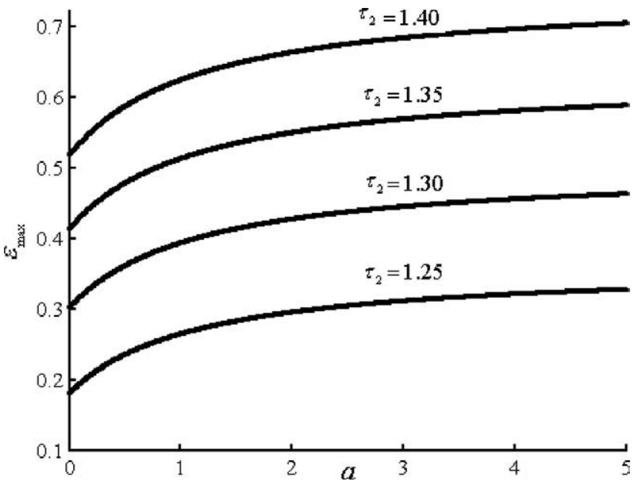


Fig. 7. The influence of τ_2 on the maximum COP $\varepsilon_{\max}$ versus a.

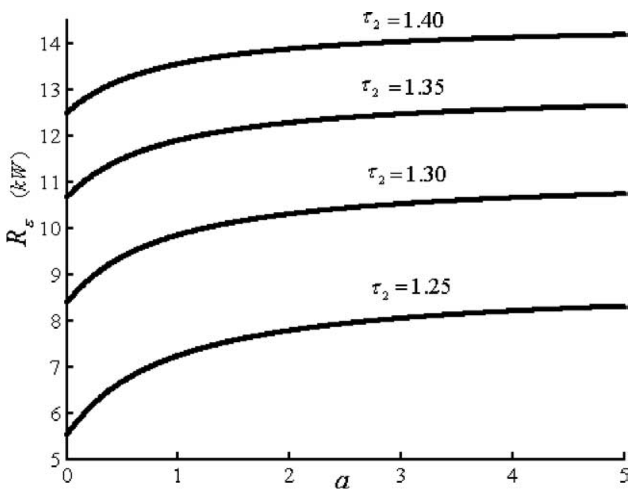


Fig. 8. The influence of τ_2 on the cooling load R_ε versus a.

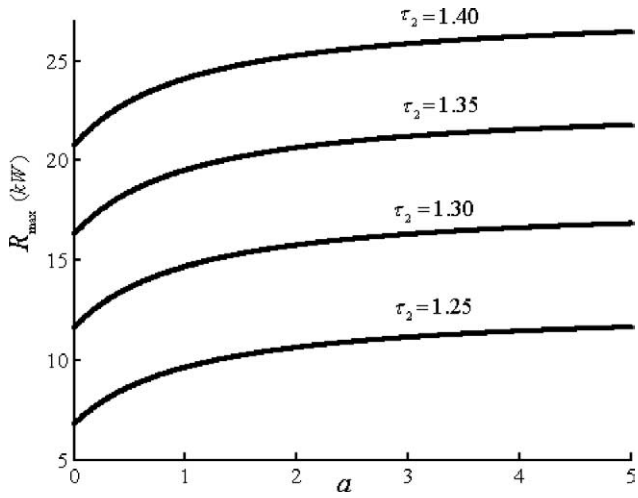


Fig. 9. The influence of τ_2 on the maximum cooling load $R_{\max}$ versus a.

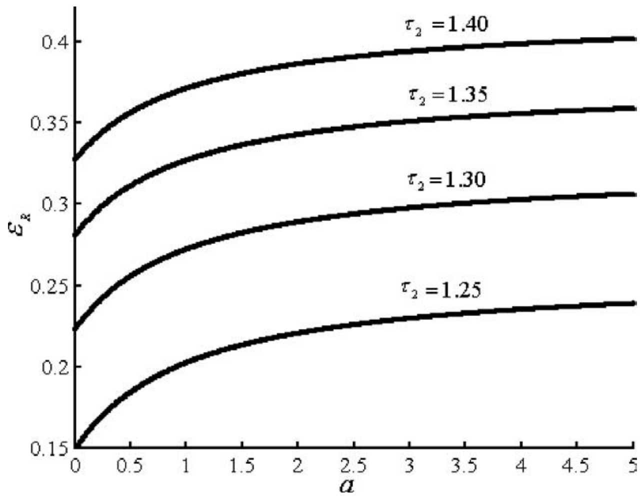


Fig. 10. The influence of τ_2 on the COP ε_R versus a.

$$R = \{UA(T_M^{-1} - b_4/a)[T_O^{-1} - (1+a)T_H^{-1} + b_4 - (2+a)(T_M^{-1} - b_4/a)]\} \times \{2(T_L^{-1} - T_H^{-1})\}^{-1} - q_L \quad (31)$$

Eliminating b_4 from Eqs. (30) and (31) yields a fundamental optimum relation of an irreversible absorption refrigerator.

4.4. When $K_L = 0$, the four-heat-reservoir irreversible absorption refrigeration cycle with linear phenomenological heat transfer law becomes the four-heat-reservoir endoreversible absorption refrigeration cycle with linear phenomenological heat transfer law. From the calculation, the $R-\varepsilon$ characteristic curve of an endoreversible absorption refrigerator is a parabola curve, i.e., there are the maximum cooling load and the corresponding COP, and when ε attains a maximum value, the corresponding cooling load is zero, as shown by curve b in Fig. 2. On the contrary, the $R-\varepsilon$ characteristic curve exhibits a loop-shape with heat leak, i.e., there are the maximum COP $\varepsilon_{\max}$ and the corresponding cooling load R_ε, the maximum cooling load $R_{\max}$ and the corresponding COP ε_R. Thus, heat leak has changed the $R-\varepsilon$ characteristic. The $R-\varepsilon$ characteristic of real absorption refrigerator curve exhibits a loop-shape, as shown in Ref. [7], therefore, it is necessary to consider the influence of heat leak on the performance besides the heat resistance loss.

4.5. The results of this paper include the optimal performance of the three-heat-reservoir irreversible refrigeration cycle with linear phenomenological heat transfer law which included heat resistance and heat leak losses, the two-heat-reservoir irreversible refrigeration cycle which included heat resistance and heat leak losses, the four-heat-reservoir endoreversible refrigeration cycle, the three-heat-reservoir endoreversible refrigeration cycle and the two-heat-reservoir endoreversible refrigeration cycle.

Case 1. $K_L = 0$.

If there is only heat resistance loss, i.e., $K_L = 0$, eliminating b_4 from Eqs. (25) and (26) yields a fundamental optimum relation of an endoreversible absorption refrigerator as follows

$$R = \{AU_4(1+a)[(1+\varepsilon^{-1})(aT_M^{-1} + T_O^{-1}) - (1+a)(T_L^{-1} + T_H^{-1}\varepsilon^{-1})]\} \times [(1+a)(y + x\varepsilon^{-1}) + (1+\varepsilon^{-1})(z+a)]^{-2} \quad (32)$$

The four-heat-reservoir irreversible refrigeration cycle with heat resistance and heat leak losses becomes the four-heat-reservoir endoreversible absorption refrigeration cycle with heat resistance loss.

Case 2. $a = 1$, $T_M = T_O$ and $U_3 = U_4$.

When $a = 1$, $T_M = T_O$ and $U_3 = U_4$, the four-heat-reservoir irreversible refrigeration cycle becomes the three-heat-reservoir irreversible refrigeration cycle with linear phenomenological heat transfer law which included heat resistance and heat leak losses, and the cooling load and the COP are

$$\varepsilon = \frac{(x+1)b_4 - T_H^{-1} - xT_M^{-1}}{T_L^{-1} + yT_M^{-1} - (y+1)b_4} \times \frac{R}{R + q_L} \quad (33)$$

$$R = \frac{2U_4A(T_M^{-1} - b_4)[(x+1)b_4 - T_H^{-1} - xT_M^{-1}]}{[(1+x)T_L^{-1} - (1+y)T_H^{-1}] + (y-x)(T_M^{-1} - b_4)} - q_L \quad (34)$$

Eliminating b_4 from Eqs. (33) and (34) yields a fundamental optimum relation of an irreversible three-heat-reservoir absorption refrigerator.

If $T_H \to \infty$ further, the four-heat-reservoir irreversible refrigeration cycle becomes the two-heat-reservoir irreversible refrigeration cycle with another linear heat transfer law which included heat resistance and heat leak losses [23,24].

Case 3. $a = 1$, $T_M = T_O$, $U_3 = U_4$ and $K_L = 0$.

If $a = 1$, $T_M = T_O$ and $U_3 = U_4$, the four-heat-reservoir irreversible refrigeration cycle becomes the three-heat-reservoir endoreversible refrigeration cycle with linear phenomenological heat transfer law which included the only loss of heat resistance [17]. The fundamental optimum relation of this refrigeration cycle is given by

$$R = \frac{AU_4[(1+\varepsilon^{-1})(T_M^{-1} + T_O^{-1}) - (T_L^{-1} + T_H^{-1}\varepsilon^{-1})]}{2[(y + x\varepsilon^{-1}) + (1+\varepsilon^{-1})]^2} \quad (35)$$

If $T_H \to \infty$ further, the four-heat-reservoir irreversible refrigeration cycle becomes the two-heat-reservoir endoreversible refrigeration cycle with linear phenomenological heat transfer law which included the only loss of heat resistance [25].

5. Conclusion

The performance of the four-heat-reservoir irreversible absorption refrigeration cycle with linear phenomenological heat transfer law, which included the heat leak from the heat sink to the cooled space and the finite-rate heat transfer, are analyzed and optimized by using finite-time thermodynamics in this paper. Moreover, the effects of the cycle parameters on the COP and the cooling load of the cycle are studied by detailed numerical examples. The ranges of the selection for the practice parameters the four-heat-reservoir irreversible absorption refrigeration cycle are derived. The results of this paper have quite commonly significance, and include the optimal performance of almost all kinds of the refrigeration cycles with linear phenomenological heat transfer law (the three-heat-reservoir irreversible refrigeration cycle, the two-heat-reservoir irreversible refrigeration cycle, the four-heat-reservoir endoreversible refrigeration cycle, the three-heat-reservoir endoreversible refrigeration cycle and the two-heat-reservoir endoreversible refrigeration cycle, see Section 4.5). Thus, the results obtain herein have realistic significance and may provide some new theoretical guidance for the optimal design and performance improvement of refrigerators.

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